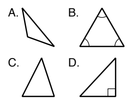
LESSON **4-1**

Practice A

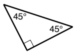
Classifying Triangles

Match the letter of the figure to the correct vocabulary word in Exercises 1–4.

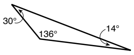


1. right triangle **D**
2. obtuse triangle **A**
3. acute triangle **B, C**
4. equiangular triangle **B**

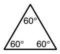
Classify each triangle by its angle measures as acute, equiangular, right, or obtuse. (*Note:* Give two classifications for Exercise 7.)

5. 

right

6. 

obtuse

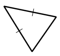
7. 

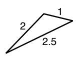
acute; equiangular

For Exercises 8–10, fill in the blanks to complete each definition.

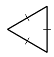
8. An isosceles triangle has **at least two** congruent sides.
9. An **equilateral** triangle has three congruent sides.
10. A **scalene** triangle has no congruent sides.

Classify each triangle by its side lengths as equilateral, isosceles, or scalene. (*Note:* Give two classifications in Exercise 13.)

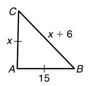
11. 
isosceles

12. 

scalene

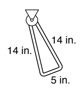
13. 
isosceles; equilateral

Find the side lengths of the triangle.



14. $AB =$ **15** $AC =$ **15** $BC =$ **21**

15. The New York City subway is known for its crowded cars. If all the seats in a car are taken, passengers must stand and steady themselves with railings or handholds. The last subway cars designed with steel hand straps were the "Redbirds" made in the late 1950s and early 1960s. The figure gives the dimensions of one of these triangular hand straps. How many hand straps could have been made from 99 inches of steel?



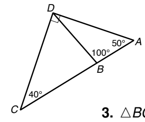
3 hand straps

LESSON **4-1**

Practice B

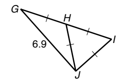
Classifying Triangles

**Classify each triangle by its angle measures.
(*Note:* Some triangles may belong to more than one class.)**



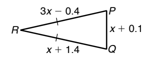
1. $\triangle ABD$ **obtuse**
2. $\triangle ADC$ **right**
3. $\triangle BCD$ **acute**

**Classify each triangle by its side lengths.
(*Note:* Some triangles may belong to more than one class.)**

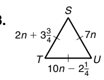


4. $\triangle GIJ$ **scalene**
5. $\triangle HIJ$ **equilateral; isosceles**
6. $\triangle GHJ$ **isosceles**

Find the side lengths of each triangle.

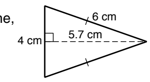
7. 

$\mathbf{PR = RQ = 2.3;\ PQ = 1}$

8. 

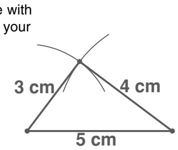
$\mathbf{ST = SU = TU = 5\frac{1}{4}}$

9. Min works in the kitchen of a catering company. Today her job is to cut whole pita bread into small triangles. Min uses a cutting machine, so every pita triangle comes out the same. The figure shows an example. Min has been told to cut 3 pita triangles for every guest. There will be 250 guests. If the pita bread she uses comes in squares with 20-centimeter sides and she doesn't waste any bread, how many squares of whole pita bread will Min have to cut up?



22 pieces of pita bread

10. Follow these instructions and use a protractor to draw a triangle with sides of 3 cm, 4 cm, and 5 cm. First draw a 5-cm segment. Set your compass to 3 cm and make an arc from one end of the 5-cm segment. Now set your compass to 4 cm and make an arc from the other end of the 5-cm segment. Mark the point where the arcs intersect. Connect this point to the ends of the 5-cm segment. Classify the triangle by sides and by angles. Use the Pythagorean Theorem to check your answer.



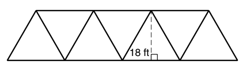
scalene, right

LESSON **4-1**

Practice C

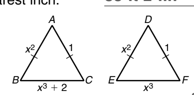
Classifying Triangles

The figure shows the side of a railway bridge. Steel girders make up the triangular pattern along the side and top. Each triangle is an equilateral triangle. In an equilateral triangle, the segment that shows the height also bisects the side it is perpendicular to. The height of the bridge is 18 feet.

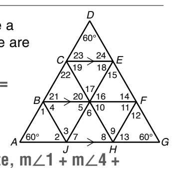


1. Find the total length of steel girder used to make this side of the bridge in feet and inches. Round to the nearest inch. **228 ft 8 in.**
2. Find the length of wooden planks needed (that is, the span of the bridge) in feet and inches. Round to the nearest inch. **83 ft 2 in.**

Use the figure for Exercises 3 and 4.



3. Find all possible values of x for each triangle. Explain why the solutions are different.
For $\triangle ABC$, $x = 1$ or -1 because the triangles are isosceles, $x^2 = 1$, so $x = \pm 1$. For $\triangle DEF$, $x \neq -1$ because a length cannot be negative, and if $x = -1$ then $EF = -1$. So $x = 1$ is the only solution for $\triangle DEF$.
4. Tell what kind of triangle each must be in all cases.
$\triangle ABC$ must be isosceles and $\triangle DEF$ must be an equilateral triangle.
5. Isosceles $\triangle GHI$ has $\overline{GH} \cong \overline{GI}$. $GH = x^2$, $GI = -2x + 15$, and $HI = -x + 4$. Find the side lengths.
$GH = GI = 25$, $HI = 9$; $GH = GI = 9$, $HI = 1$
6. Given that $\overline{AG} \parallel \overline{BF} \parallel \overline{CE}$ and $m\angle A = m\angle D = m\angle G = 60°$, write a paragraph proof proving that all the numbered angles in the figure are congruent. (*Hint:* Mark off each angle as you go.)



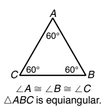
Possible answer: By the Corr. Angles Postulate, $m\angle A = m\angle 21 = m\angle 23 = 60°$ and $m\angle G = m\angle 14 = m\angle 24 = 60°$. Construct a line parallel to $\overline{CE}$ through D. Then also by the Corr. Angles Postulate, $m\angle D = m\angle 22 = m\angle 1 = m\angle 15 = m\angle 12 = 60°$. By the definition of a straight angle and the Angle Add. Postulate, $m\angle 1 + m\angle 4 + m\angle 21 = 180°$, but $m\angle 1 = m\angle 21 = m\angle A = 60°$. Therefore by substitution and the Subt. Prop. of Equality, $m\angle 4 = 60°$. Similar reasoning will prove that $m\angle 11 = m\angle 18 = m\angle 19 = 60°$. By the Alt. Int. Angles Theorem, $m\angle 19 = m\angle 20$ and $m\angle 18 = m\angle 16$. $m\angle 20 = m\angle 10$ and $m\angle 16 = m\angle 5$ by the Vertical Angles Theorem. By the Alt. Int. Angles Theorem, $m\angle 4 = m\angle 2$ and $m\angle 5 = m\angle 7$ and $m\angle 10 = m\angle 8$ and $m\angle 11 = m\angle 13$. By the definition of a straight angle, the Angle Addition Postulate, substitution, and the Subt. Prop., $m\angle 17 = m\angle 6 = m\angle 3 = m\angle 9$. Substitution will show that the measure of every angle is 60°. Because every angle has the same measure, all of the angles are congruent by the definition of congruent angles.

LESSON **4-1**

Reteach

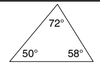
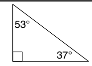
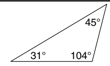
Classifying Triangles

You can classify triangles by their angle measures. An **equiangular triangle,** for example, is a triangle with three congruent angles.

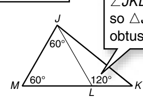


$\angle A \cong \angle B \cong \angle C$
$\triangle ABC$ is equiangular.

Examples of three other triangle classifications are shown in the table.

Acute Triangle	Right Triangle	Obtuse Triangle
 72°, 50°, 58°	 53°, 37°	 45°, 31°, 104°
all acute angles	one right angle	one obtuse angle

You can use angle measures to classify $\triangle JML$ at right.

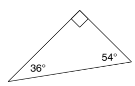


$\angle JLM$ and $\angle JLK$ form a linear pair, so they are supplementary.

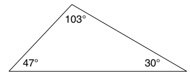
$m\angle JLM + m\angle JLK = 180°$	Def. of supp. ∠s
$m\angle JLM + 120° = 180°$	Substitution
$m\angle JLM = 60°$	Subtract.

Since all the angles in $\triangle JLM$ are congruent, $\triangle JLM$ is an equiangular triangle.

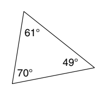
Classify each triangle by its angle measures.

1. 

right

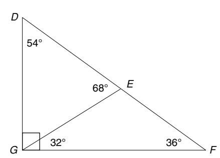
2. 

obtuse

3. 

acute

Use the figure to classify each triangle by its angle measures.

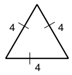
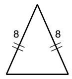
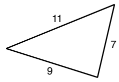


4. $\triangle DFG$ **right**
5. $\triangle DEG$ **acute**
6. $\triangle EFG$ **obtuse**

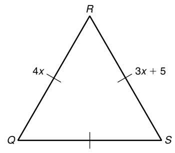
LESSON 4-1

Reteach
Classifying Triangles continued

You can also classify triangles by their side lengths.

Equilateral Triangle	Isosceles Triangle	Scalene Triangle
 all sides congruent	 at least two sides congruent	 no sides congruent

You can use triangle classification to find the side lengths of a triangle.



Step 1 Find the value of x.

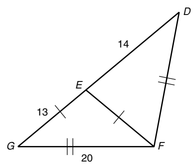
$QR = RS$	Def. of ≅ segs.
$4x = 3x + 5$	Substitution
$x = 5$	Simplify.

Step 2 Use substitution to find the length of a side.

$4x = 4(5)$	Substitute 5 for x.
$= 20$	Simplify.

Each side length of $\triangle QRS$ is 20.

Classify each triangle by its side lengths.

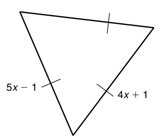


7. $\triangle EGF$ isosceles

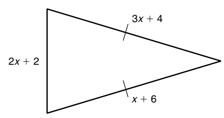
8. $\triangle DEF$ scalene

9. $\triangle DFG$ isosceles

Find the side lengths of each triangle.

10. 

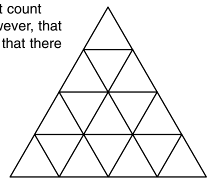
9; 9; 9

11. 

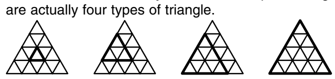
7; 7; 4

LESSON 4-1

Challenge
A Tantalizing Triangle



How many triangles are in the figure at right? Many people just count the small triangles and decide that there are sixteen in all. However, that answer accounts for only the small "one-part" triangles. Notice that there are actually four types of triangle.



one-part triangles | four-part triangles | nine-part triangles | a sixteen-part triangle

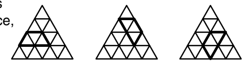
Refer to the figure at right above. How many triangles of each type are there?

1. one-part triangles 16
2. four-part triangles 7
3. nine-part triangles 3
4. sixteen-part triangles 1

5. Use your answers to Exercises 1–4. What is the total number of triangles in the figure? 27

6. What is the total number of *triangle midsegments* in the figure? 21

You may have noticed that there are several other types of geometric shapes within the figure above. For instance, the diagrams at right highlight just three of the many "three-part trapezoids" that can be found within it.



Refer to the figure above, and use the counting techniques developed in Exercises 1–6.

7. What is the total number of trapezoids in the figure? 27, 12, 3, 9, 3, 3 = 57

8. What is the total number of *trapezoid midsegments* in the figure? 12

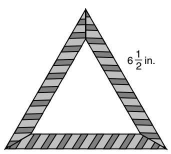
9. Assume that all the small triangles are equilateral. What is the total number of rhombuses in the figure above? 18, 3 = 21

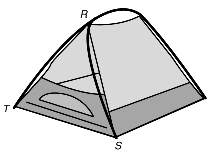
10. What is the total number of parallelograms in the figure above? 3, 18, 3, 9, 3 = 36

11. Identify three types of polygons that can be found in the figure above other than those named in Exercises 1–10. Your polygons may be convex or concave. Sketch each polygon in the space at right. Then find the total number of each type of polygon that you identified in the figure. 3, 3, 3, 9 Answers will vary.

LESSON 4-1

Problem Solving
Classifying Triangles



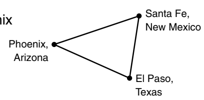


1. Aisha makes triangular picture frames by gluing three pieces of wood together in the shape of an equilateral triangle and covering the wood with ribbon. Each side of a frame is $6\frac{1}{2}$ inches long. How many frames can she cover with 2 yards of ribbon?
3 frames

2. A tent's entrance is in the shape of an isosceles triangle in which $\overline{RT} \cong \overline{RS}$. The length of $\overline{TS}$ is 1.2 times the length of a side. The perimeter of the entrance is 14 feet. Find each side length.
$4\frac{3}{8}$ ft; $4\frac{3}{8}$ ft; $5\frac{1}{4}$ ft

Use the figure and the following information for Exercises 3 and 4.

The distance "as the crow flies" between Santa Fe and Phoenix is 609 kilometers. This is 245 kilometers less than twice the distance between Santa Fe and El Paso. Phoenix is 48 kilometers closer to El Paso than it is to Santa Fe.

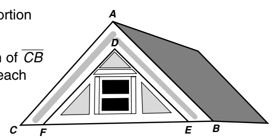


3. What is the distance between each pair of cities?
Santa Fe and El Paso, 427 km; El Paso and Phoenix, 561 km; Phoenix and Santa Fe, 609 km

4. Classify the triangle that connects the cities by its side lengths. scalene

Choose the best answer.

A *gable*, as shown in the diagram, is the triangular portion of a wall between a sloping roof.



5. Triangle *ABC* is an isosceles triangle. The length of $\overline{CB}$ is 12 feet 4 inches and the congruent sides are each $\frac{3}{4}$ this length. What is the perimeter of $\triangle ABC$?

A 31 ft 4 in. C 21 ft 7 in.
B 30 ft 10 in. D 18 ft 6 in.

6. In $\triangle DEF$, $\overline{DE}$ and $\overline{DF}$ are each 6 feet 3 inches long. This length is 0.75 times the length of $\overline{FE}$. What is the perimeter of $\triangle DEF$?

F 12 ft 4 in. H 17 ft 2 in.
G 14 ft 7 in. J 20 ft 10 in.

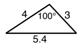
LESSON 4-1

Reading Strategies
Vocabulary Development

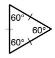
The table below shows seven ways to classify different triangles, by angle measures and by side lengths. Remember, you cannot simply assume things about segment lengths and angle measures. Information must be given in writing or by marks and labels in the diagram.

Classification	Description	Example
acute triangle	triangle that has *three* acute **angles**	78° 69° 33°
equiangular triangle	triangle that has *three* congruent acute **angles**	
right triangle	triangle that has *one* right **angle**	
obtuse triangle	triangle that has *one* obtuse **angle**	120°
equilateral triangle	triangle with *three* congruent **sides**	
isosceles triangle	triangle that has *at least two* congruent **sides**	
scalene triangle	triangle that has *no* congruent **sides**	3 5 4

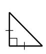
Classify the following triangles by side lengths and angle measures. There will be more than one answer.

1. 

scalene, obtuse

2. 

equilateral, equiangular, acute

3. 
isosceles, right

Suppose you are asked to draw a triangle by using the given information. If you think it is possible to draw such a triangle, classify the triangle. Otherwise write *no such triangle*.

4. $\triangle ABC$ with $AB = 3$, $BC = 3$, and $CA = 5$ isosceles triangle

5. $\triangle XOZ$ with $m\angle X = 92°$, $m\angle O = 92°$, and $m\angle Z = 27°$ no such triangle

6. $\triangle MNK$ with $m\angle M = 90°$, $m\angle N = 60°$, and $m\angle K = 30°$ scalene right triangle